Journal of Fluids and Structures (1999) **13,** 153–158
Article No.: jfls.1998.0187
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BRIEF NOTE

A NOTE ON PHASE-LOCKED STATES AT LOW REYNOLDS NUMBERS

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(Received 17 August 1997 and in revised form 21 August 1998)

In this note, extensive phase-locked states in the low Reynolds number cylinder wake are presented. Results of the present experimental investigation have expanded the scope of previous work in the subharmonic and superharmonic ranges. The search and discussion for these phase-locked states were carried out within the framework of nonlinear dynamical systems.

1. INTRODUCTION

An aero/hydroelastic system consisting of a cylinder under the forced excitation of various frequencies and amplitudes in an imposed unidirectional flow of an unbounded domain has been the subject of considerable research in the past three decades. Assessments of the advances in the understanding of many aspects of this coupled cylinder-wake response can be found in the overviews presented by Mair & Maull (1971), Sarpkaya (1979), and Bearman (1984). It has been known that the vortex shedding frequency f_s^* can lock onto the cylinder oscillation frequency f_e and to multiples and submultiples of the cylinder oscillation frequency. These are the well-established concepts of the commonly called primary (or fundamental), subharmonic and superharmonic lock-on, respectively. Besides the primary lock-on, which has attracted the most attention, due primarily to its significant engineering applications, there have been three cases of phase-locked states ($f_e/f_s^* = 1/2$, 1/3 and 1/4) observed in the subharmonic range, and two cases of phase-locked states ($f_e/f_s^* = 2$ and 3) observed in the superharmonic range (Stansby 1976; Durgin, March & Lefebvre 1980; Ongoren & Rockwell 1988; Williamson & Roshko 1988).

If one could investigate the aforementioned phase-locking phenomena from the perspective of nonlinear dynamical systems, it can readily be seen that, in general, they are resonant responses which can exist in systems of coupled oscillators or oscillators coupled to periodic external excitations. Following the terminology of nonlinear dynamical systems the ratio $\Omega = f_e/f_s$ is defined as the uncoupled winding number and $\omega = f_e/f_s^*$ as the coupled winding number, where f_s is the inherent vortex-wake formation frequency without external excitation, and f_s^* is the coupled wake oscillation frequency. Note that in the presence of nonlinear coupling, the natural system oscillation frequency f_s will shift to f_s^*. Basically, phase locking can occur whenever the frequency of a harmonic of f_e approaches some harmonic of f_s^* and the frequencies of the two oscillators lock exactly into a rational value f_e/f_s^*. For a specific resonant state, the system displays a "resonant" type of region in the A–Ω plane, where $A(=a/D$, D being the cylinder diameter) is the nondimensionalized cylinder oscillation amplitude. For a stationary cylinder, the vortex formation and shedding process is a function of the Reynolds number in the low Reynolds number range. It can be expected that f_s^*, and thus the phase-locked state, in addition to the cylinder excitation frequency and amplitude, is also a function of the Reynolds number.

Olinger & Sreenivasan (1988) were the first to study this coupled cylinder-wake response within the context of a low-order dynamic system. About 30 resonant horns in the region $\Omega \leqq 1$ for Re $= 64$–76 had been noted, with ten of them having resonable width, including the primary phase-locked states. These phase-locked diagrams are included in Figure 1. Their findings had apparently suggested a very complicated and busy picture of possible body-wake interactions inherent in the present system. One obvious implication associated with such a result is that in the common practice of scanning across the A–Ω plane with discrete increments of A and Ω to study the wake characteristics at various resonant and/or nonresonant states, great care must be exercised in selecting proper intervals for ΔA and $\Delta\Omega$ so as to have the necessary resolution. If one could selectively vary A and/or Ω in a continuous manner, within certain limited ranges in the A–Ω plane, intuitively, one would expect to be able to detect any reasonably fine wake structure within those ranges. This could easily be realized by, say, oscillation of a flexible cable in a uniform flow (in such a case, Ω is fixed and A is a continuous variable), or oscillation of a rigid cylinder in a shear flow (A is fixed and Ω is a continuous variable with limited variations in Reynolds number), or oscillation of a cable in a shear flow (both A and Ω are variables with limited variations in Reynolds number). Nevertheless, one complex issue arises in these approaches. It is the secondary flow effect in the body-wake interaction mechanism, which has to be properly assessed (Woo, Cermak & Peterka 1989) before any results thus obtained can be used in a nominally two-dimensional flow situation.

In a study of flexible cables with a smooth surface forced to oscillate at the first mode of various frequencies and antinode amplitudes in a shear flow with linear velocity distribution, we discovered that, due to the crossing of several resonant horns in the A–Ω plane, the vortex wake behind the cables tended to break down into a number of discrete resonant cells, each with a corresponding frequency (Woo 1998). Within the range $0{\cdot}5 < \Omega \leqq 1{\cdot}5$, besides the $\omega = 3/5, 2/3, 3/4$ and $1/1$ phase-locked states that have been reported by Olinger & Sreenivassan (1988), 13 additional phase-locked states with $\omega = 4/7, 5/7, 7/9, 4/5, 5/6, 6/7, 7/6, 6/5, 5/4, 9/7, 4/3, 7/5$ and $3/2$ have been identified. These additional states were the basis for the mapping of the resonant boundaries in uniform flow in this study.

2. EXPERIMENTAL METHODS

The phase diagrams of these newly identified phase-locked states in the A–Ω plane were obtained with a rigid cylinder 0·5 cm in diameter in uniform flow. The wind tunnel used was an open-return type which had a cross-section of 30×30 cm and a test-section of 150 cm in length. The turbulence level was of the order of 0·1%.

The test-cylinder had an effective length (between end plates) of 20 cm. The end plates were properly manipulated to ensure parallel vortex shedding. Oscillation of the cylinder was provided using an electronic shaker and the oscillation amplitude was monitored with a fibre optic vibration probe. A miniature hot-film probe, placed at $3{\cdot}0D$ downstream

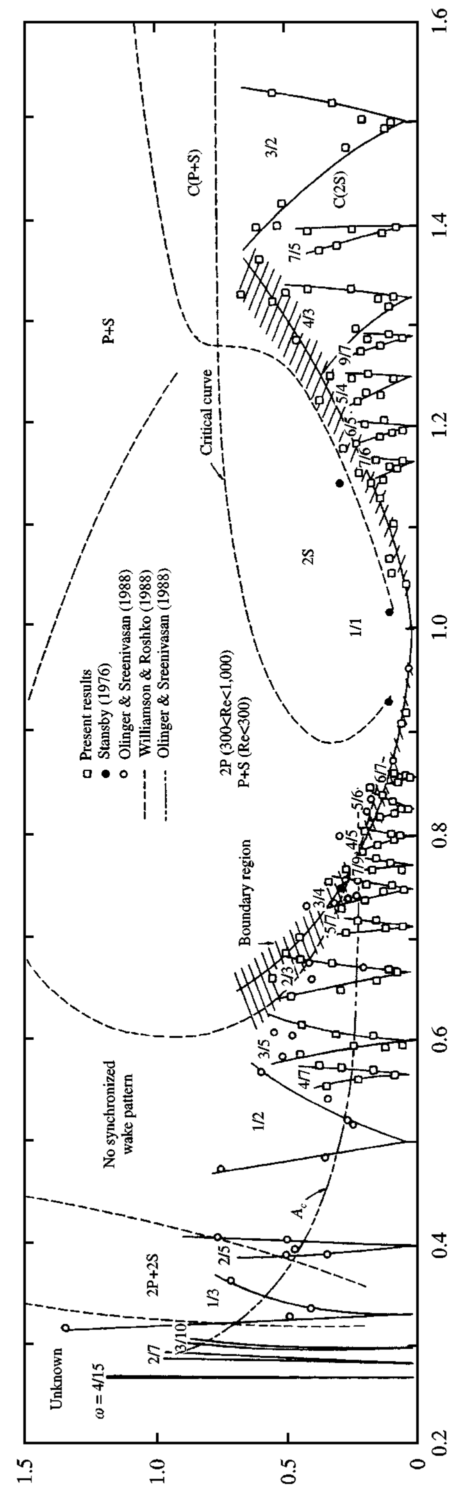

Figure 1. Phase-locked states and map of vortex synchronization patterns.

of the test cylinder and $2{\cdot}0D$ away from the wake centreline, was used for measuring the wake frequency content. The frequencies of the cylinder oscillation and the wake were determined with a spectrum analyzer. The test Reynolds number based on cylinder diameter was 420–435.

3. RESULTS AND DISCUSSION

Figure 1 shows all of the phase-locked states presently known in the region $0{\cdot}26 < \Omega \leqq 1{\cdot}5$. According to a series of experimental observations by Williamson & Roshko (1988) for $\mathrm{Re} \leqq 1000$, oscillations of the cylinder can generate a number of classes of wake structure, each with distinct geometrical characteristics. Their map of the vortex synchronization pattern, with critical curves marking the transition from one vortex shedding pattern to another, is superimposed on the phase-locked diagram and shown in Figure 1.

In Figure 1, following the notations given by Williamson & Roshko, "P" represents a vortex pair, "S" represents a single vortex, and "C" represents coalescence. The richness and complexity of the nonlinear dynamical phenomena existing in the present aeroelastic system can clearly be seen. It should be noted that because the absolute instability of the vortex wake [see, e.g., Monkewitz & Nguyen (1987)] is receptive to perturbations of finite amplitude only, none of the tips of the resonant horns touch the Ω-axis.

As the cylinder oscillation amplitude increases, the possible range of a phase-locked states also increases. This results in a widening of the resonant horn and consequently the possible overlapping of several neighbouring resonant horns. One of the prominent features of Figure 1 is the sheer dominance of the primary lock-on state in the A–Ω plane. As a result, there exist two rather extensive boundary regions on both sides of the primary lock-on state, where overlaps with a series of other phase-locked states have taken place. It can readily be seen that at the oscillation amplitude level of $A = 0{\cdot}1$, the overlaps include the $\omega = 5/6$ (i.e. $f_s^*/f_e = 1{\cdot}2$) and 7/6 ($f_s^*/f_e = 0{\cdot}86$) resonant horns. At the $A = 0{\cdot}29$ level, the overlaps include the $\omega = 3/4$ ($f_s^*/f_e = 1{\cdot}33$) and 4/3 ($f_s^*/f_e = 0{\cdot}75$) resonant horns. Despite the difference in Reynolds numbers, vortex shedding in these overlapped regions are, respectively, characterized by the gradients of the first and second broken lines in Figure 2(a, b) given by Stansby (1976).

For the $\omega = 1/1$ and 3/4 neighbouring phase-locked states, we have observed that depending upon the position within the overlap region, the degree of vortex-wake mode competition which leads to mode uncertainty varies (Ciliberto & Gollub 1984; Jensen, Bak & Bohr 1984). Consequently, the wake can be in a state of slow undulatory oscillation, or changing abruptly and intermittently between the 1/1 and 3/4 modes, or in a state of chaotic oscillation. These results should have provided explanations to the findings of Stansby (1976), that there are ranges on both sides of the primary lock-on where the "unforced vortex sheddings" (note this is a clear misnomer) are more stable in frequency than the corresponding vortex shedding for the stationary cylinder, and f_s^*/f_s is approximately proportional to f_e/f_s. This shifting of f_s^* at the boundaries of the primary lock-on, from the values found when the cylinder is stationary, can also be seen in some of the earlier results presented by Bearman & Davies (1977) and Bearman & Obasaju (1982).

In the subharmonic range in Figure 1, the line of critical amplitude A_c was given by Olinger & Sreenivasan (1988), based on the results of iteration of the sine circle map. Below the critical line, the vortex wake only exhibits periodic and quasi-periodic behaviour. Above the critical line, due to widening of the resonant horns and the overlaps and competitions of possible multiple phase-locked states, a universal transition to chaotic oscillation of the wake occurs. This could be the underlying reason why Ongoren & Rockwell (1988) were able to detect the $\omega = 1/2$ phase-locked state at a lower oscillation amplitude of $A = 0{\cdot}13$

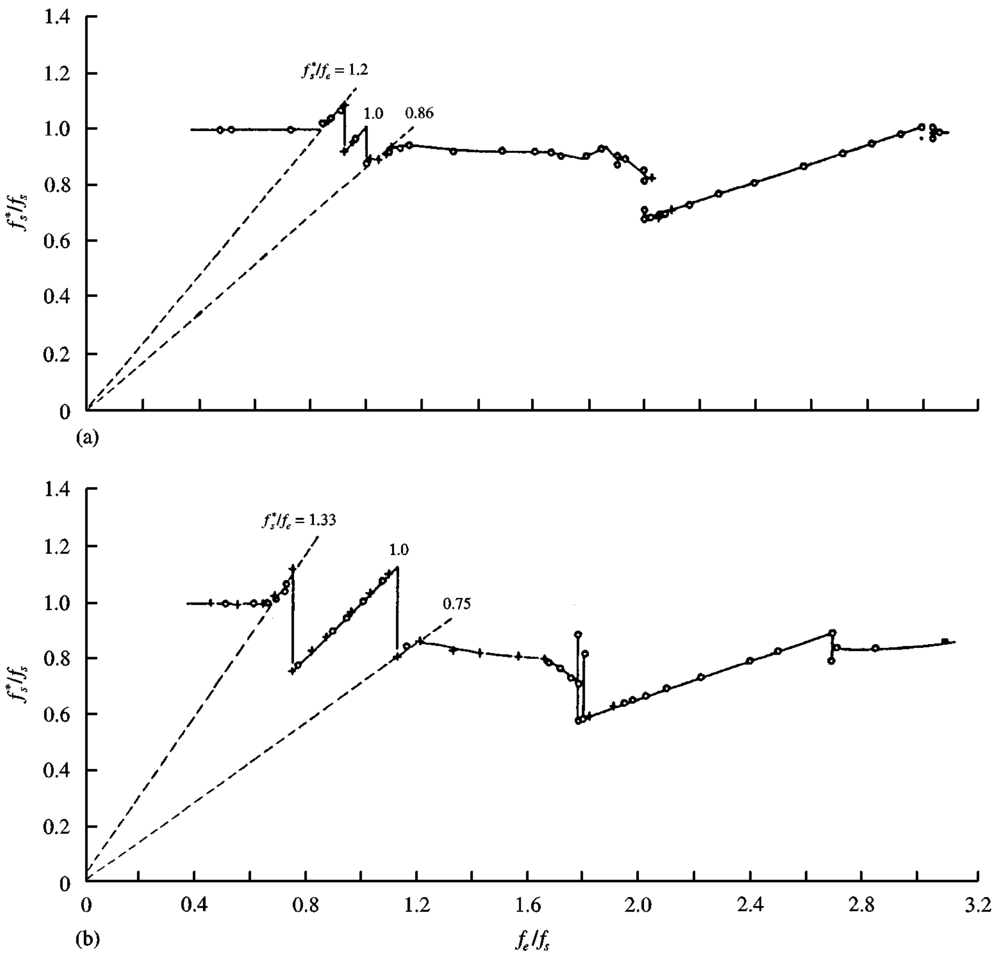

Figure 2. The variation of f_s^*/f_s with f_e/f_s in uniform flow with (a) $a/D = 0{\cdot}10$ and (b) $a/D = 0{\cdot}29$ for Re $\approx$ 9200 ($0 < f_e/f_s < 2{\cdot}3$) and 7000 ($f_e/f_s > 2{\cdot}3$) : ○, increasing cylinder frequency; +, decreasing cylinder frequency [from Stansby (1976)].

(Re $\simeq$ 855), while Williamson & Roshko (1988), who presumably conducted most of their experiments at $A > 0{\cdot}2$, had failed to detect it and had also categorically concluded that no synchronized wake pattern was observed in the $2/5 \leqq \omega < 1/1$ zone. It is interesting to note that for the $\omega = 1/3$ and part of the $\omega = 2/5$ resonant horns, even far above the A_c line Williamson & Roshko were still able to observe an orderly wake pattern in the 2P + 2S mode. Furthermore, above the primary lock-on in the superharmonic region, for oscillation amplitude A, approximately under 1·0, despite the possible overlaps of several phase-locked states, according to Williamson & Roshko, the wake is in an orderly coalescence mode (or the so called "recovery mode" by Ongoren & Rockwell 1988). A closer investigation into the causes of these obvious differences is currently under study.

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